

## REPORT DOCUMENTATION PAGE

Form Approved  
OMB No. 0704-0188

AD-A232 764

is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this burden to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Street, Washington, DC 20503.

REPORT DATE		3. REPORT TYPE AND DATES COVERED Final Report, 1 Jun 86 to 31 May 90	
4. TITLE AND SUBTITLE PARALLEL SOLUTION OF VERY LARGE SPARSE LINEAR PROGRAMS		5. FUNDING NUMBERS AFOSR-86-0172 61102F 2304/A8	
6. AUTHOR(S) Olvi L. Mangasarian			
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Computer Sciences Department University of Wisconsin Madison, WI 53706		8. PERFORMING ORGANIZATION REPORT NUMBER AFOSR-86-0172	
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) AFOSR/MM Bldg 410 Bolling AFB DC 20332-6448		10. SPONSORING / MONITORING AGENCY REPORT NUMBER AFOSR-TR-91-0145	
11. SUPPLEMENTARY NOTES		DTIC ELECTE MAR 08 1991 S E D	
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution unlimited.		12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  The principal area of research was the parallel solution of large sparse linear programs. A number of new algorithms with increased performance were developed and tested for special cases.			
14. SUBJECT TERMS		15. NUMBER OF PAGES	
		16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT SAR

Final  
Technical Report  
to  
Air Force Office of Scientific Research

on

Grant No. AFOSR-86-0172:

"Parallel Solution of Very Large  
Sparse Linear Programs"

Principal Investigator:  
Olvi L. Mangasarian

Computer Sciences Department  
University of Wisconsin  
Madison, WI 53706  
(608) 262-1204  
[olvi@cs.wisc.edu](mailto:olvi@cs.wisc.edu)

Accession For	
NTIS	GRA&I <input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	23



December 6, 1989

## **Contents**

---

1. Summary of Achievements	1
2. Summary of Technical Results	2
3. Papers Written under Support of Grant AFOSR-86-0172	4
4. Ph.D. Dissertations Supervised	12

## **1. Summary of Achievements**

---

During the 41-month period June 1, 1986 - November 30, 1989, the following was achieved under support from Grant AFOSR-86-0172:

- (i) Seventeen Computer Sciences Department technical reports were written and submitted for outside publication. Five have already appeared in print, three have been accepted for publication and nine are under review.
- (ii) Four Ph.D. dissertations were supervised. Two dissertations are complete, the third will be completed in June 1990 and the fourth in December 1991.
- (iii) Forty-one technical talks were presented by the Principal Investigator outside the University of Wisconsin at scientific meetings and other universities on research work supported by this grant.

## 2. Summary of Technical Results

Seventeen reports were written in the 41-month period beginning June 1, 1986, under support from Grant AFOSR-86-0172. Titles and abstracts of these reports are given in Section 3 of this report.

The principal area of research undertaken under this grant is the parallel solution of very large sparse linear programs. The ground work for this approach was laid in [1] where the inherently serial successive overrelaxation (SOR) scheme for solving very large sparse linear programs was parallelized. With the serial version of the SOR scheme [5] we were able to solve linear programs with as many as 125,000 variables and 500,000 constraints on a MicroVax II in less than 72 hours. We are not aware of any general linear programming packages that can solve linear programs of this size. The parallel approach proposed in [1] required a relaxation factor  $\omega$  to be restricted to the interval  $0 < \omega \frac{2}{1+\sigma}$  where  $\sigma$  is a positive number depending on the coupling among the parallel blocks of the problem. If the problem is loosely coupled,  $\sigma$  approaches zero. In [2, 5] a more efficient parallel SOR scheme was proposed in which  $\omega$  is restricted to the classical interval  $0 < \omega < 2$ . This resulted in larger step-sizes and a faster algorithm. Speedup efficiency of the algorithm as tested on the CRYSTAL token-ring multicomputer and the Sequent Balance 21,000 multiprocessor was in the 65 to 100 percent range. Another interesting and useful result was that a serial implementation of our parallel SOR scheme was more efficient than the original serial SOR scheme.

In [3] a new and practically useful characterization of the solution set of a convex program was obtained. In particular we have shown that the gradient of the objective function of a convex program is constant on the solution set of the problem. Furthermore, the solution set lies in an affine subspace orthogonal to this constant gradient, and is in fact in the intersection of this affine subspace with the feasible region. This result can be used to give simple and computationally useful characterizations of solution sets of a number of important problems.

In [4] methods are proposed for the solution of the non-monotone (hence non-convex and very difficult) complementarity problem. A guaranteed lower bound to a least 2-norm solution of this problem is given.

In [6] a parallel asynchronous SOR is proposed for the solution of symmetric linear complementarity problems and linear programs. In [7] a two-stage (SOR) method for solving the symmetric linear complementarity problem is given that can solve large problems that cannot be tackled by existing algorithms. In [8] a parallel implementation of Lemke's algorithm is given for solving the linear complementarity problem with speedup efficiency of essentially 100%.

In [9] new asynchronous algorithms are proposed for the solution of the symmetric linear complementarity problems tested on the Sequent Symmetry S81 multiprocessor with speedup efficiency in the 43%-91% range. The test results clearly show the superiority of the asynchronous algorithms over their synchronous counterparts.

In [10] error bounds are given which guarantee how far a given arbitrary point is from the solution set of a nondegenerate monotone linear complementarity problem. Nondegeneracy induces a simplicity in the bounds that cannot be obtained otherwise. These bounds are important theoretically and useful computationally.

In [11] we establish convergence of an asynchronous parallel algorithm for solving the nonsymmetric linear complementarity problem. Numerical results show that asynchronism improves computational efficiency.

In [12] we show that under appropriate conditions, perturbing a convex programming objective function is equivalent to minimizing the perturbation function on the solution set of the original unperturbed convex program. This result has useful computational implications.

In [13] we use linear programming to diagnose breast cancer data. A computer program based on our work is currently in use at University of Wisconsin Hospitals, and has successfully diagnosed 98% of new cases encountered.

In [14] we compare our linear programming approach of [13] with a neural network approach on breast cancer data. The results indicate superiority of the linear programming approach.

In [15] we establish a practical results which shows that under a nondegeneracy assumption, linearizing the objective function around a point sufficiently close to a solution, will yield an exact solution in one step. This leads to finite termination of any convergent algorithm if this linearization is performed periodically.

In [16] we show that complexity of the pattern recognition problem varies from being NP-complete to being in  $P$ , depending on the norm employed. This result has real practical computational significance.

In [17] we establish convergence and test an interior proximal-point algorithm for linear programming. On a standard set of test problems it outperforms MINOS 5.0 by a factor of 5.

### **3. Papers Written under Support of Grant AFOSR-86-0172**

- [1] O. L. Mangasarian & R. De Leone: "Parallel successive overrelaxation methods for symmetric linear complementarity problems and linear programs", *Journal of Optimization Theory and Applications* 54, 1987, 437-446.
- [2] O. L. Mangasarian & R. De Leone: "Parallel gradient projection successive overrelaxation for symmetric linear complementarity problems and linear programs", *Annals of Operations Research* 14, 1988, 41-59.
- [3] O. L. Mangasarian: "A simple characterization of solution sets of convex programs", *Operations Research Letters* 7, 1988, 21-26.
- [4] O. L. Mangasarian: "Least norm solution of non-monotone linear complementarity problems", *UW Comp. Sci. Tech. Rept. 686*, February 1987, to appear, *Kantorovich Memorial Volume*, American Mathematical Society, Providence, RI, 1988.
- [5] R. De Leone & O. L. Mangasarian: "Serial and parallel solution of large scale linear programs by augmented Lagrangian successive overrelaxation", in A. Kurzhanski, K. Neumann & D. Pallaschke (eds.): "Optimization, parallel processing and applications", *Lecture Notes in Economics and Mathematical Systems Volume 304*, Springer-Verlag, Berlin 1988, 103-124.
- [6] Karen M. Thompson: "A parallel asynchronous successive overrelaxation algorithm for solving the linear complementarity problem", *University of Wisconsin Tech. Rept. 705*, July 1987, submitted to *SIAM Journal on Matrix Analysis and Applications*.
- [7] Karen M. Thompson: "A two-stage successive overrelaxation for solving the linear complementarity problem", *University of Wisconsin Computer Sciences Tech. Rept. 706*, July 1987, submitted to *Mathematical Programming*.
- [8] Karen M. Thompson: "A parallel pivotal algorithm for solving the linear complementarity problem", *University of Wisconsin Computer Sciences Tech. Rept. 707*, July 1987, submitted to *Journal of Optimization Theory and Applications*.
- [9] R. De Leone & O. L. Mangasarian: "Asynchronous parallel successive overrelaxation for the symmetric linear complementarity problem", *Mathematical Programming Series B*, 42, 1988, 347-361.

- [10] O. L. Mangasarian: "Error bounds for nondegenerate monotone linear complementarity problems", University of Wisconsin Computer Sciences Tech. Rept. 787, August 1988, to appear in Mathematical Programming Series B.
- [11] R. De Leone, O. L. Mangasarian & T.-H. Shieu: "Multi-sweep asynchronous parallel successive overrelaxation for the nonsymmetric linear complementarity problem", University of Wisconsin Computer Sciences Tech. Rept. 792, September 1988, to appear in Annals of Operations Research 22, 1989.
- [12] M. C. Ferris & O. L. Mangasarian: "Finite perturbation of convex programs", University of Wisconsin Computer Sciences Tech. Rept. 802, October 1988, submitted to Applied Mathematics & Optimization.
- [13] W. H. Wolberg & O. L. Mangasarian: "Multisurface method of pattern separation applied to breast cytology diagnosis", submitted to Proceedings of the National Academy of Sciences U.S.A.
- [14] W. H. Wolberg & O. L. Mangasarian: "Computer-aided diagnosis of breast cytology via two expert systems", submitted to Acta Cytologica.
- [15] M. C. Ferris & O. L. Mangasarian: "Minimum principle sufficiency", University of Wisconsin Computer Sciences Tech. Rept. 853, June 1989, submitted to Mathematical Programming Series A.
- [16] O. L. Mangasarian, R. Setiono & W. H. Wolberg: "Pattern recognition via linear programming: Theory and application to medical diagnosis", University of Wisconsin Tech. Rept. 878, September 1989, submitted to SIAM Workshop on Large-Scale Numerical Optimization, Cornell University, Ithaca, New York, October 19-20, 1989.
- [17] R. Setiono: "An interior dual proximal point algorithm for linear programs", University of Wisconsin Tech. Rept. 879, September 1989, submitted to ORSA Journal on Computing.

Reproduced below are abstracts of the above reports given in the same order as the above listing:

1. A parallel successive overrelaxation (SOR) method is proposed for the solution of the fundamental symmetric linear complementarity problem. Convergence is established under a relaxation factor which approaches the classical value of 2 for a loosely coupled problem. The parallel SOR approach is then applied to solve the symmetric linear complementarity problem associated with the least norm solution of a linear program.

2. Gradient projection successive overrelaxation (GP-SOR) algorithm is proposed for the solution of symmetric linear complementarity problems and linear programs. A key distinguishing feature of this algorithm is that when appropriately parallelized, the relaxation factor interval  $(0, 2)$  is not reduced. In a previously proposed parallel SOR schemes the substantially reduced relaxation interval mandated by the coupling terms of the problem often led to slow convergence. The proposed parallel algorithms are applied to finding the least 2-norm solution of linear programs. Efficiency of the algorithm is in the 50 to 100 percent range as demonstrated by computational results on the CRYSTAL token-ring multicomputer and the Sequent Balance 21000 multiprocessor.

3. By means of elementary arguments we first show that the gradient of the objective function of a convex program is constant on the solution set of the problem. Furthermore the solution set lies in an affine subspace orthogonal to this constant gradient, and is in fact in the intersection of this affine subspace with the feasible region. As a consequence we give a simple polyhedral characterization of the solution set of a convex quadratic program and that of a monotone linear complementarity problem. For these two problems we can also characterize a priori the boundedness of their solution sets without knowing any solution point. Finally we give an extension to nonsmooth convex optimization by showing that the intersection of the subdifferentials of the objective function on the solution set is nonempty and equals the subdifferential of the objective function at any point in the relative interior of the optimal solution set. In addition, the solution set lies in the intersection with the feasible region of an affine subspace orthogonal to some subgradient of the objective function at a relative interior point of the optimal solution set.

4. We show that a least 2-norm solution of a general linear complementarity problem (LCP) can be obtained by solving a sequence of perturbed quadratic programs. The norm of the solutions of the perturbed problems approaches monotonically from below the norm of a least 2-norm solution of the LCP, as the perturbation parameter approaches zero. For sufficiently large value of the perturbation, the quadratic program is strongly convex and easily solved by Lemke's method. This yields a guaranteed lower bound to the norm of a least 2-norm solution. For some LCP's, even non-monotone ones, the perturbed quadratic program may give a solution to the LCP for a finite value of the perturbation parameter. For monotone LCP's the perturbed quadratic program yields the least 2-norm solution for a finite value of the perturbation parameter if and only if the least 2-norm solution of the linearized LCP equals that of the original LCP.

5. Serial and parallel successive overrelaxation (SOR) methods are proposed for the solution of the augmented Lagrangian formulation of the dual of a linear program. With the proposed serial version of the method we have solved linear programs with as many as 125,000 constraints and 500,000 variables in less than 72 hours on a MicroVax II. A parallel implementation of the method was carried out on a Sequent Balance 21000 multiprocessor with speedup efficiency of over 65% for problem sizes of up to 10,000 constraints, 40,000 variables and 1,400,000 nonzero matrix elements.

6. We present a parallel asynchronous successive overrelaxation algorithm for the solution of symmetric linear complementarity problems and linear programs. A distinguishing feature of this algorithm is that processors need not communicate after each update of the solution vector and therefore processor idle time can be avoided. The proposed parallel algorithm is applied to finding least 2-norm solutions of linear programs. Improvement is observed over the synchronized version of the algorithm, the parallel gradient projection successive overrelaxation algorithm.

7. We propose a two-stage successive overrelaxation (SOR) algorithm for solving the symmetric linear complementarity problem. After the first SOR preprocessing stage this algorithm concentrates on updating a certain prescribed subset of variables which is determined by exploiting the complementarity property. We demonstrate that this algorithm successfully solves problems with as many as 10,000 variables which cannot be tackled by other current algorithms.

8. We propose a parallel implementation of the classical Lemke's algorithm for solving the linear complementarity problem. The algorithm is designed for a loosely coupled network of computers which is characterized by relatively high communication costs. We provide an accurate prediction of speedup based on a simple operation count. The algorithm produces speedups near  $p$ , where  $p$  is the number of processors, when tested on large problems as demonstrated by computational results on the CRYSTAL token-ring multicomputer and the Sequent Balance 21000 multiprocessor.

9. Convergence is established for asynchronous parallel successive overrelaxation (SOR) algorithms for the symmetric linear complementarity problem. For the case of a strictly diagonally dominant matrix convergence is achieved for a relaxation factor interval of  $(0, 2]$  with line search, and  $(0, 1]$  without line search. Computational tests on the Sequent Symmetry S81 multiprocessor give speedup efficiency in the 43%-91% range for the cases for which convergence is established. The tests also show superiority of the asynchronous SOR algorithms over their synchronous counterparts.

10. Error bounds and upper Lipschitz continuity results are given for monotone linear complementarity problems with a nondegenerate solution. The existence of a nondegenerate solution considerably simplifies the error bounds compared with problems for which all solutions are degenerate. Thus when a point satisfies the linear inequalities of a nondegenerate complementarity problem, the residual that bounds the distance from a solution point consists of the complementarity condition alone, whereas for degenerate problems this residual cannot bound the distance to a solution without adding the square root of the complementarity condition to it. This and other simplified results are a consequence of the polyhedral characterization of the solution set as the intersection of the feasible region  $\{z | Mz + q \geq 0, z \geq 0\}$  with a single linear affine inequality constraint.

11. Convergence is established for the multi-sweep asynchronous parallel successive overrelaxation (SOR) algorithm for the nonsymmetric linear complementarity problem. The algorithm was originally introduced in [4] for the symmetric linear complementarity problem. Computational tests show the superiority of the multi-sweep asynchronous SOR algorithm over its single-sweep counterpart on both symmetric and nonsymmetric linear complementarity problems.

12. This paper concerns a characterization of the finite perturbation property of a convex program. When this property holds, finite perturbation of the objective function of a convex program leads to a solution of the original problem which minimizes the perturbation function over the set of solutions of the original problem. This generalizes a finite termination property of the proximal point algorithm for linear programs and characterizes finite Tikhonov regularization of convex programs.

13. Multisurface pattern separation is a mathematical method for distinguishing between elements of two pattern sets. Each element of the pattern set is comprised of various scalar observations. In this paper, we use the diagnosis of breast cytology to demonstrate the applicability of this method to medical diagnosis and decision making. Eleven cytological characteristics of breast fine-needle aspirates (fna) reported to differ between benign and malignant samples were graded (1 to 10) at the time of sample collection. Nine characteristics were found to differ significantly between benign and malignant. Mathematically, these values for each sample were represented by a point in a nine-dimensional space of real variables. Benign points were separated from malignant ones by planes determined by linear programming. Correct separation was accomplished in 369 of 370 samples (201 benign and 169 malignant). In the one misclassified malignant case, the fna cytology was so definitely benign and the cytology of the excised cancer so definitely malignant that we believe the tumor was missed on aspiration. This mathematical method is applicable to other medical diagnostic and decision-making problems.

14. Two computer-driven expert systems were trained to correctly diagnose 369 fine-needle breast aspirates (fna's) on the basis of nine cytological descriptive parameters and were tested on 24 newly obtained fna's (eighteen benign and six malignant). One system, generated by multisurface pattern separation, correctly classified all training and test samples, while the other, generated by a connectionist algorithm (neural network), incorrectly classified one of the benign test samples as malignant (i.e. one false positive). Even though the multisurface expert system appears better able to diagnose new data patterns than the neural network system, both expert systems aid breast cytological diagnosis and can serve as models for other applications.

15. We characterize the property of obtaining a solution to a convex program by minimizing over the feasible region a linearization of the objective function at any of its solution points (Minimum Principle Sufficiency). For the case of a monotone linear complementarity problem this MPS property is completely equivalent to the existence of a nondegenerate solution to the problem. For the case of a convex quadratic program the MPS property is equivalent to the span of the Hessian of the objective function being contained in the normal cone to the feasible region at any solution point plus the cone generated by the gradient of the objective function at any solution point. This in turn is equivalent to the quadratic program having a weak sharp minimum. An important application of the MPS property is that minimizing on the feasible region a linearization of the objective function at a point in a neighborhood of a solution point gives an exact solution of the convex program. This leads to finite termination of convergent algorithms that periodically minimize such a linearization.

16. A decision problem associated with a fundamental nonconvex model for linearly inseparable pattern sets is shown to be NP-complete. Another nonconvex model that employs an  $\infty$ -norm instead of the 2-norm, can be solved in polynomial time by solving  $2n$  linear programs, where  $n$  is the (usually small) dimensionality of the pattern space. An effective LP-based finite algorithm is proposed for solving the latter model. The algorithm is employed to obtain a nonconvex piecewise-linear function for separating points representing measurements made on fine needle aspirates taken from benign and malignant human breasts. A computer program trained on 369 samples has correctly diagnosed each of 48 new samples encountered and is currently in use at the University of Wisconsin Hospitals.

17. An interior point algorithm for obtaining a proximal point solution of a linear program is presented. Results from our implementation of this algorithm have been very encouraging. For 36 test problems including 32 NETLIB problems, we obtain a total time speedup of 5.6 over the MINOS 5.0 simplex package. We also describe an implementation of our algorithm for linear programs with upper-bounded variables, such as the multicommodity Patient Distribution System models of the Military Airlift Command. We have been able to solve some of these multicommodity problems with 8-figure accuracy and speedup of as much as 24 over the MINOS 5.0. Furthermore our run times on the Astronautics ZS-1 are comparable with those of AT&T's KORBX times for some of the problems.

**4. Ph.D. Dissertations Supervised**

<b>Student</b>	<b>Dissertation Title</b>	<b>Completion Date</b>
Michael C. Ferris	Weak sharp minima and penalty functions mathematical programming	August 1989
Lois Brady	Condition constants for solutions of convex inequalities	August 1988
Rudy Setiono	Interior penalty methods for linear and quadratic programs	June 1990
Kristin Bennett	Pattern recognition via mathematical programming	December 1991